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Non-zero nature of Theta 13 - generating Neutrino Reactor mixing angle via perturbation ($\boldsymbol{\theta_{13}}$)

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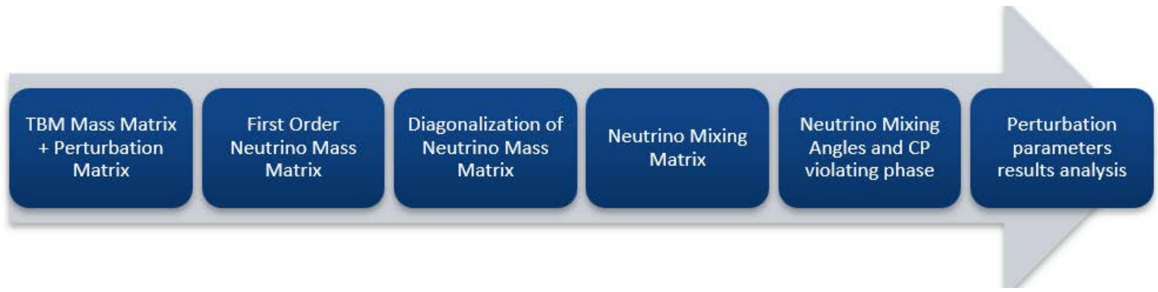
Article

ABSTRACT



Thermonuclear fusion reactions happening at the core of the sun lead to the creation of a huge amount of energy and a large number of neutrinos. At very high temperatures and pressure, two protons come and fuse to each other leading to the creation of a deuteron which consists of one proton and one neutron, it means two protons not only fuse together but also a beta decay happens in which a proton gets converted into neutron leading to the emission of a positron and an electron neutrino. This is part of a brooder proton-proton cycle of reactions which is one of the dominant cycles of nuclear fusion reaction that happens at the core of the sun. Additionally, the Standard Model of Particle Physics, which proposes the massless and chargeless character of the neutrino as being sufficient until neutrino oscillation happens, presents the idea of elementary particles by taking into account three fundamental forces. The possibility of a neutrino having a mass other than zero surfaces in neutrino oscillation. Prior to now, Homestake's experiment, which became the solution to the solar neutrino puzzle, gave us the first indication of neutrino oscillation by showing that the Earth's neutrino flux did not match that predicted by the traditional solar model. It is essentially the difference between the flux of neutrinos empirically detected on Earth and the flux of neutrinos theoretically anticipated to be released from the sun. The solution to the solar neutrino problem was proposed using neutrino oscillations. The Tri-Bimaximal Mixing Ansatz provided a clear explanation for previous experimental findings on neutrino oscillations. The disappearing reactor angle theta - 13 was one of the main predictions of the TBM ansatz. The TBM texture is no longer experimentally viable due to the findings of the Daya Bay, RENO, Double Chooz, MINOS, and T2K investigations. These reveal to us theta – 13's non-zero character. To create the non-vanishing reactor angle, we attempt to investigate the perturbations of various TBM terms while continuing to employ TBM as the leading order matrix in the current work. We gravitate towards first-order perturbations.

Keywords: Neutrino oscillation, PMNS matrix, TBM matrix, Perturbation

INTRODUCTION

Experiments on cosmic rays provide us the proof for the existence of number of particles other than electron, proton, neutrons and photons.[1,2] By using high-energy accelerators and detectors a large number of new particles were produced and detected in nuclear reactions that decayed rapidly after being created in high energy collision between other particles.[3] It is predicted that there are more than 200 elementary particles discovered so far.[4] Elementary in the sense that they are structureless. These are analyzed in terms of their mass, intrinsic spin magnetic moment and interaction properties. These particles are classified as bosons and fermions which were further classified as massless bosons, mesons, leptons and baryons. Among these elementary particles, we are going to discuss the Neutrino - the Ghost Particle belonging to lapton sector of fermions.[5] Moreover, the thermonuclear fusion reactions happening at the core of the sun lead to the creation of a huge amount of energy and a large number of neutrinos.[6] At very high temperature and pressure, two protons come and fuse to each other leading to the creation of a deuteron which consists of one proton and one neutron, it means two protons

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not only fuse together but also a beta decay happens in which a proton gets converted into neutron leading to the emission of a positron and an electron neutrino. This is part of a brooder proton-proton cycle of reactions which is one of the dominant cycles of nuclear fusion reaction that happens at the core of the sun

$$p + p \rightarrow H_1^2 + e^+ + \vartheta_e$$

The Standard Model of Particle Physics, which proposes the existence of elementary particles by taking into account three fundamental forces, suggests that neutrinos possess neither mass nor charge, which is sufficient until neutrino oscillation occurs. The possibility of a neutrino having a mass other than zero surfaces in neutrino oscillation. When the measured flow of neutrinos discovered on Earth does not match the observed flux of neutrinos indicated by the traditional solar model,[7] Homestake's experiment gave us our first indication of neutrino oscillation.[8] This discovery became the solution to the solar neutrino puzzle. It is essentially the disparity between the flux of neutrinos anticipated by theory to be released by the sun and the flux of neutrinos actually measured on Earth. The solution to the solar neutrino problem was proposed using neutrino oscillations.[9] The atmospheric neutrino oscillation and the solar neutrino issue were both detected, and both have been shown to be compatible with the tribimaximal form of the mixing matrix U of the lepton sector. The Tri-Bimaximal Mixing Ansatz provided a clear explanation for previous experimental findings on neutrino oscillations.[10] The disappearing reactor angle theta - 13 was one of the main predictions of the TBM ansatz. A special postulate form for the PMNS laptop matrix U called the "tribimaximal matrix" takes all the components in square moduli form as

$$\begin{bmatrix} |U_{e_1}|^2 & |U_{e_2}|^2 & |U_{e_3}|^2 \\ |U_{\mu_1}|^2 & |U_{\mu_2}|^2 & |U_{\mu_3}|^2 \\ |U_{\tau_1}|^2 & |U_{\tau_2}|^2 & |U_{\tau_3}|^2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$$

$$\therefore U^0 = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \quad \text{---(1)}$$

Currently, experiments at the level of 5σ prevent such mixing. It is evident to us in this matrix that the third mixing angle θ_{13} is entirely disappearing. Tribimaximal mixing angle θ_{13} should be assumed to be zero, which was the case prior to the experimental findings from the 127-day Daya Bay collaborative exhibition and the 229-day RENO data,[10] which revealed that angle θ_{13} was not zero. An earlier generation of neutrino oscillation experiments employed this form of the matrix as a zeroth-order approximation to more broad forms of the PMNS matrix that are also consistent with the available data. Tribimaximal mixing in the PMNS matrix can be identified using lepton mixing angles, namely

$\delta = 0,\ \theta_{31} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35.3450\ ,\ \theta_{23} = 0$ and $\theta_{13} = 0.$

The TBM texture is no longer experimentally viable due to the finding of the Daya Bay, RENO, Double Chooz, MINOS, and T2K experiments. These reveal to us theta -13's non-zero character. Experimentally, it was discovered that $\theta_{13} \approx 8.5$ was significant, refuting the aforementioned prediction. In comparison to other neutrino mixing angles, the non-zero value of mixing angle θ_{13} is minuscule. Come to the conclusion that the observed neutrino mixing cannot be explained by the tribimaximal matrix operation. The diminutive size of θ_{13}in comparison to θ_{23} and θ_{31}, the other two mixing angles, encourages us to consider a minor tweak on the tribimaximal structure that results in a plausible neutrino mixing matrix. In flavor basis charged lepton mass matrix is diagonal. Moreover, if M^0 is the mass matrix satisfying tribimaximal mixing and m_1, m_2, m_3 are the left-handed neutrino Majorana masses, then

$$M^0 = U^0 \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} (U^0)^T$$

Using equation (1), we get

$$M^0 = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}$$

$$M^0 = \begin{bmatrix} \frac{2m_1 + m_2}{3} & \frac{m_2 - m_1}{3} & \frac{m_1 - m_2}{3} \\ \frac{m_2 - m_1}{3} & \frac{m_1 + 2m_2 + 3m_3}{6} & \frac{-m_1 + 2m_2 + 3m_3}{6} \\ \frac{m_1 - m_2}{3} & \frac{-m_1 - 2m_2 + 3m_3}{6} & \frac{m_1 + 2m_2 + 3m_3}{6} \end{bmatrix}$$

Let us assume that $m_0 = \left(\frac{m_1+m_2+m_3}{3}\right)$, $\Delta_{32} \cong (m_3 - m_2)$ *and* $\Delta_{31} \cong (m_3 - m_1)$. Here, by diagonal phase transformation, the complex mass eigenvalues m_1, m_2, m_3 can be converted to positive and real, i.e., $D = diag.\left(e^{i\lambda_1}, e^{i\lambda_2}, 1\right)$; where λ_i are the Majorana phases which do not influence neutrino oscillations.

$$\therefore M^0 = \begin{bmatrix} m_0 - \frac{\Delta_{31}}{3} & \frac{\Delta_{31} - \Delta_{32}}{3} & -\frac{\Delta_{31} - \Delta_{32}}{3} \\ \frac{\Delta_{31} - \Delta_{32}}{3} & m_0 + \frac{\Delta_{31}}{6} & \frac{\Delta_{31} + 2\Delta_{32}}{6} \\ -\frac{\Delta_{31} - \Delta_{32}}{3} & \frac{\Delta_{31} + 2\Delta_{32}}{6} & m_0 + \frac{\Delta_{31}}{6} \end{bmatrix}$$

Now, because of the fact that $|\Delta_{32}| >> \Delta_{31} \equiv (m_2 - m_1)$, we can approximate $\Delta_{32} \approx \Delta_{31} \equiv \Delta$. Here Δ sets the scale for atmospheric neutrino oscillations. We can make use of such limits in the flavor basis to unperturbed the mass matrix as;

$$M^0 \approx \begin{bmatrix} m_0 - \frac{\Delta}{3} & 0 & 0 \\ 0 & m_0 + \frac{\Delta}{6} & \frac{\Delta}{6} \\ 0 & \frac{\Delta}{2} & m_0 + \frac{\Delta}{6} \end{bmatrix}$$

As of right now, the solar mass splitting is also absent, with now $m_1^{(0)} = m_2^{(0)} = m_0 - \frac{\Delta}{3}$ and $m_3^{(0)} = m_0 + \frac{2\Delta}{3}$. Now that $\theta_{13} \neq 0$ has been established, our goal is to create this splitting using the same perturbation Hamiltonian.

For this, we assume that $m_1^{(0)}$, $m_2^{(0)}$ *and* $m_3^{(0)}$ are real and positive numbers. Here, M^0 is the component of the neutrino mass matrix that most clearly emerges from the fundamental model, and it is regarded as such. Our goal is to show how M^0 differs from other models that were obtained from the mixing matrix's tribimaximal form through a particular technique. This may be achieved by parameterizing Pontecorvo, Maki, Nakagawa, and Sakata (PMNS) by three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and a single-phase angle known as δ_{CP} (complex phase) associated C.P violation as follows.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\,\delta_{CP}} \\ -s_{23}s_{13}c_{12}e^{i\,\delta_{CP}} & -s_{23}s_{13}s_{12}e^{\,i\delta_{CP}} + c_{23}s_{12} & s_{23}c_{13} \\ -c_{23}s_{13}c_{12}e^{i\,\delta_{CP}} + s_{23}s_{12} & -c_{23}s_{13}s_{12}e^{i\,\delta_{CP}} - s_{23}c_{12} & c_{23}c_{13} \end{bmatrix}$$

It is obvious that the tribimaximal mixing matrix's $U_{e_3}^0$, assumed as zero, is really non-zero. For many scholars, the role of non-vanishing $U_{e_3}^0$ or equivalently θ_{13} becomes the entryway. Its non-zero character is crucial for the absence of CP non-conservation in neutrino oscillations and might be the breakthrough in the quest to understand leptogenesis. Even though the mixing angles of the two sectors are very different, the non-zero character of θ_{13} will be identical to the quark sector, where mixing across all three generations and CP violation is a proven outcome. In the event of a CP violation, both the complicated phase δ_{CP} and θ_{13} might have non-vanishing.

PERTURBATION

There have been many attempts to generate the non-zero nature of θ_{13},, but we prefer to use the perturbation approach, specifically the first order perturbation, to identify the structure of the Majorana Mass matrix $M = M^0 + M'$, where $M' << M^0$, in order to obtain θ_{13} and solar mass splitting where M^0 and M' will be complex and symmetric. However, M^0 is Hermitian or real and symmetric, in the tribimaximal mixing form. These two situations must now be handled individually. Additionally, the unperturbed mass eigenstates for M^0 in the mass basis are as follows:

$$\psi_1^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \psi_2^{(0)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \psi_3^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

where the basis vectors $\psi_1^{(0)}$and $\psi_2^{(0)}$are conventional, degenerate, and chosen to accurately recreate the solar mixing. These eigenstates are only the column of U^0 in terms of the flavour basis, which may be expressed as:

$$\psi_1^{(0)} = \begin{pmatrix} \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{6}} \end{pmatrix}, \quad \psi_2^{(0)} = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} \end{pmatrix}, \quad \psi_3^{(0)} = \begin{pmatrix} 0 \\ \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \end{pmatrix}$$

FIRST ORDER PERTURBATION

First, we assume that M' is symmetric, real and thus Hermitian, allowing us to generate a non-zero θ_{13} without encountering a CP violation, which results in δ = 0. Moreover by consider a physical system with a Hamiltonian M^0 matching a free system with an eigen basis of $|\psi_3^{(0)}>$, where $|\psi_3^{(0)}>$ is the eigen ket of M^0 with an eigen value of $m_0^{(0)}$such that

$$M^0|\psi_3^{(0)}> = m_0{}^{(0)}|\psi_3{}^{(0)}> \qquad \text{(Time dependent Schrodinger's Equation)}$$

To first order we have,

$$\psi_3 = \psi_3^{(0)} + \sum_{n\neq 3} C_{n3}\psi_n^{(0)}$$

We know that, by Orthonormality property

$$<\psi_3^{(0)}|\psi_j^{(0)}> = \delta_{3j} \quad \text{where,} \quad \delta_{3j} = 0 \text{ for } j = 3$$
$$= 1 \text{ for } j \neq 3$$

Also, by Completeness property;

$$\sum n\,|\psi_3^{(0)}><\psi_3^{(0)}| = 1$$

Now, we slightly altered the Majorana mass to make it $M = M^0 + \lambda M'$; $0 \le \lambda \le 1$. The dominance of perturbation is shown by the perturbation constant λ.

Let's assume that the new eigenvalue and eigenfunction are $M|\psi_3> = m_0|\psi_3>$, where, $|\psi_3>$
(3rd new state of the entire Hamiltonian) = $|\psi_3^{(0)}>$ (3rd old basis)+ $|\Delta\psi_{j>}$ (same quantity that can be extended in terms of old basis). Therefore, since the perturbation has no effect on the system's Hilbert space, the previous basis set can be kept.

When we multiply eigen value m_0 by the power series in terms of λ and expand eigen function $|\psi_3>$, we obtain

$$\psi_3> = \left|\psi_3^{(0)}> + \lambda\right|\psi_3^{(1)}> + \lambda^2\left|\psi_3^{(2)}> + \lambda^3\right|\psi_3^{(3)}>$$
$$+ \lambda^4|\psi_3^{(4)}> + - - -$$

and. $\quad m_0 = m_0^{(0)} + \lambda m_0^{(1)} + \lambda^2 m_0^{(2)} + \lambda^3 m_0^{(3)} + \lambda^4 m_0^{(4)} + - - -$

But $M|\psi_3> = m_0|\psi_3>$ and $M = M^0 + \lambda M'$; $0 \le \lambda \le 1$

Therefore, above equation can be rewrite as

$$(M^0+\lambda M')\left(|\psi_3^{(0)}>+\lambda|\psi_3^{(1)}>+\lambda^2|\psi_3^{(2)}>+\lambda^3|\psi_3^{(3)}>+\lambda^4|\psi_3^{(4)}>+---\right)=(m_0^{(0)}+\lambda m_0^{(1)}+\lambda^2 m_0^{(2)}+\lambda^3 m_0^{(3)}+\lambda^4 m_0^{(4)}+---)(|\psi_3^{(0)}>+\lambda|\psi_3^{(1)}>+\lambda^2|\psi_3^{(2)}>+\lambda^3|\psi_3^{(3)}>+\lambda^4|\psi_3^{(4)}>+---)$$

$$\Rightarrow M^0\left|\psi_3^{(0)}>+M^0\lambda\right|\psi_3^{(1)}>+M^0\lambda^2\left|\psi_3^{(2)}>+M^0\lambda^3\right|\psi_3^{(3)}>+M^0\lambda^4\left|\psi_3^{(4)}>+\lambda M'\right|\psi_3^{(0)}>+\lambda^2 M'\left|\psi_3^{(1)}>+\lambda^3 M'\right|\psi_3^{(2)}>+\lambda^4 M'\left|\psi_3^{(3)}>+\lambda^5 M'\right|\psi_3^{(4)}>=(m_0^{(0)}|\psi_3^{(0)}>+m_0^{(0)}\lambda|\psi_3^{(1)}>+m_0^{(0)}\lambda^2|\psi_3^{(2)}>+m_0^{(0)}\lambda^3|\psi_3^{(3)}>+m_0^{(0)}\lambda^4|\psi_3^{(4)}>+\lambda m_0^{(1)}|\psi_3^{(0)}>+\lambda^2 m_0^{(1)}|\psi_3^{(1)}>+\lambda^3 m_0^{(1)}|\psi_3^{(2)}>+\lambda^4 m_0^{(1)}|\psi_3^{(3)}>+\lambda^5 m_0^{(1)}|\psi_3^{(4)}>+\lambda^2 m_0^{(2)}|\psi_3^{(0)}>+\lambda^3 m_0^{(2)}|\psi_3^{(1)}>+\lambda^4 m_0^{(2)}|\psi_3^{(2)}>+\lambda^5 m_0^{(2)}|\psi_3^{(3)}>+\lambda^6 m_0^{(2)}|\psi_3^{(4)}>+---$$

$$\Rightarrow M^o\lambda^1|\psi_3^{(1)}>+M^0\lambda^2\left|\psi_3^{(2)}>+M^0\lambda^3\right|\psi_3^{(3)}>+M^0\lambda^4\left|\psi_3^{(4)}>+\lambda M'\right|\psi_3^{(0)}+\lambda^2 M'\left|\psi_3^{(1)}>+\lambda^3 M'\right|\psi_3^{(2)}>+\lambda^4 M'\left|\psi_3^{(3)}>+\lambda^5 M'\right|\psi_3^{(4)}>=m_0^{(0)}\lambda|\psi_3^{(1)}>+m_0^{(0)}\lambda^2|\psi_3^{(2)}>+m_0^{(0)}\lambda^3|\psi_3^{(3)}>+m_0^{(0)}\lambda^4|\psi_3^{(4)}>+\lambda m_0^{(1)}|\psi_3^{(0)}>+\lambda^2 m_0^{(1)}|\psi_3^{(1)}>+\lambda^3 m_0^{(1)}|\psi_3^{(2)}>+\lambda^4 m_0^{(1)}|\psi_3^{(3)}>+\lambda^5 m_0^{(1)}|\psi_3^{(4)}>+\lambda^2 m_0^{(2)}|\psi_3^{(0)}>+\lambda^3 m_0^{(2)}|\psi_3^{(1)}>+\lambda^4 m_0^{(2)}|\psi_3^{(2)}>+\lambda^5 m_0^{(2)}|\psi_3^{(3)}>+\lambda^6 m_0^{(2)}|\psi_3^{(4)}>+---$$

Here, the first order correction to the zeroth eigenvalue in this case is represented by $m_0^{(1)}$, the first order correction to the zeroth eigen function by $|\psi_3^{(1)}>$, the second order correction by $m_0^{(2)}$ and $|\psi_3^{(2)}>$, and so on.

Now, let us equate the power of λ from both sides, we get

$$\lambda^0;\ M_0|\psi_3^{(0)}>=\ m_0^{(0)}|\psi_3^{(0)}>$$

$$\lambda^1;\ M^o|\psi_3^{(1)}>+M'|\psi_3^{(0)}>=\ m_0^{(0)}|\psi_3^{(1)}>+m_0^{(1)}|\psi_3^{(0)}>$$

$$\lambda^2;M^0|\psi_3^{(2)}>+M'|\psi_3^{(1)}>=\ m_0^{(0)}|\psi_3^{(2)}>+m_0^{(1)}|\psi_3^{(1)}>+m_0^{(2)}|\psi_3^{(0)}>$$

$$\lambda^3;\ M^0|\psi_3^{(3)}>+M'|\psi_3^{(2)}>=\ m_0^{(0)}|\psi_3^{(3)}>+m_0^{(1)}|\psi_3^{(2)}>+m_0^{(2)}|\psi_3^{(1)}>$$

$$\lambda^4;\ M^0|\psi_3^{(4)}+M'|\psi_3^{(3)}>=m_0^{(0)}|\psi_3^{(4)}>+m_0^{(1)}|\psi_3^{(3)}>+m_0^{(2)}|\psi_3^{(2)}>$$

Let us rewrite $|\psi_3^{(1)}>$ as $|\psi_3^{(1)}>=1|\psi_3^{(1)}>$

Now by using Completeness property;

$$1=\ \textstyle\sum_n|\psi_3^{(0)}><\psi_3^{(0)}|$$

$$|\psi_3^{(1)}>=\ \textstyle\sum_n\left|\psi_3^{(0)}><\psi_3^{(0)}\right|\psi_3^{(1)}>$$

Let us consider $<\psi_3^{(0)}|\psi_3^{(1)}>=\ C_{n3}^{(1)}$

$$\Rightarrow|\psi_3^{(1)}>=\sum_n C_{n3}^{(1)}\,|\psi_3^{(0)}>$$

Also, $|\psi_3^{(2)}>=\ 1|\psi_3^{(2)}>$

$$\Rightarrow|\psi_3^{(2)}>=\sum_n\left|\psi_3^{(0)}><\psi_3^{(0)}\right|\psi_3^{(2)}>$$

$and\ |\psi_3^{(2)}>=\sum_n C_{n3}^{(2)}\,|\psi_3^{(0)}>\ and$ so on

In general, $\psi_3\ =\ \psi_3^{(0)}+\sum_{n\neq3}C_{n3}\psi_n^{(0)}$

Moreover, $M^0|\psi_3^{(0)}>=\ m_0{}^{(0)}|\psi_3{}^{(0)}>$

$\therefore M^0|\psi_3^{(1)}>=\ m_3^{(0)}\sum_n C_{n3}^{(1)}\,|\psi_3^{(0)}>$

We receive the first order correction as

$$M^0|\psi_3^{(1)}>+M'|\psi_3^{(0)}>=\ m_0^{(0)}|\psi_3^{(1)}>+m_0^{(1)}|\psi_3^{(0)}>$$

$$\therefore m_3^{(0)}\textstyle\sum_n C_{n3}^{(1)}\,|\psi_3^{(0)}>+M'|\psi_3^{(0)}>=m_0^{(0)}\sum_n C_{n3}^{(1)}\,|\psi_3^{(0)}>+m_0^{(1)}|\psi_3^{(0)}>$$

$$\Rightarrow\ m_3^{(0)}\sum_n C_{n3}^{(1)}\,|\psi_3^{(0)}>-m_0^{(0)}\sum_n C_{n3}^{(1)}\,|\psi_3^{(0)}>=\ -M'|\psi_3^{(0)}>+m_0^{(1)}|\psi_3^{(0)}>$$

$$\sum_n C_{n3}^{(1)}\,(m_3^{(0)}-\ m_0{}^{(0)})\,|\psi_3^{(0)}>=-(M'-m_0^{(1)})|\psi_3^{(0)}>$$

$$\sum_n C_{n3}^{(1)}\,(m_3^{(0)}-\ m_0{}^{(0)})\,|\psi_3^{(0)}>+(M'-m_0^{(1)})|\psi_3^{(0)}>=0$$

Let us consider $m_0^{(0)}=m_n^{(0)}$ and multiply the above equation by $<\psi_n^{(0)}|$ from the right side, we get

$$<\psi_n^{(0)}|\sum_n C_{n3}^{(1)}\,(m_3^{(0)}-\ m_n{}^{(0)})\,|\psi_n^{(0)}>+<\psi_n^{(0)}|(M'-m_0^{(1)})|\psi_3^{(0)}>=0$$

$$\sum_n C_{n3}^{(1)}(m_3^{(0)}-\ m_n{}^{(0)})\ <\psi_n^{(0)}|\psi_n^{(0)}>+<\psi_n^{(0)}|M'|\psi_3^{(0)}>-m_0^{(1)}<\psi_n^{(0)}|M'|\psi_3^{(0)}>=0$$

Using the property: $\sum_n f(n)\delta_{n3}=\sum_{n=1}^{3} f(n)\delta_{n,2}=f(2)$, we get

$$C_{n3}^{(1)}\left(m_3^{(0)}-m_n^{(0)}\right)+<\psi_n^{(0)}|M'|\psi_3^{(0)}>-m_0^{(1)}\delta_{n3}=0$$

For n= 3

$$<\psi_n^{(0)}|M'|\psi_3^{(0)}>=M'=m_0^{(1)}$$

For n ≠ 3

$$C_{n3}^{(1)}(m_3^{(0)}-m_n^{(0)})+<\psi_n^{(0)}|M'|\psi_3^{(0)}>=0$$

$$C_{n3}^{(1)}=-\frac{<\psi_0^{(0)}|M'|\psi_3^{(0)}>}{m_3^{(0)}-m_n^{(0)}}=-C_{3n}^{(1)}\ (n\neq3)$$

Here the coefficient C_{n3} is real.

Discussion

In this study, we have tried to generate the non - vanishing reactor mixing angle θ_{13} by preserving most of the predictions of TBM ansatz of the neutrino mass matrix. For this we have taken the TBM mass matrix as the leading order matrix and then perturb this matrix using standardized non relativistic quantum mechanics perturbation. Our ultimate aim was to generate the θ_{13} and try to relate the perturbation parameter with the deviation of atmospheric mixing angle θ_{12} from its maximal value. Finally, we have found that the maximum reactor angle that we can generate using perturbation is 2 degree which is way below the experimental value. Hence, we conclude that in our case the perturbation of TBM mass matrix does not lead to the experimentally allowed value of θ_{13}.

Conclusion

We conclude by demonstrating that the TBM ansatz has a non-zero neutrino mixing reactor angle $\boldsymbol{\theta_{13}}$ and is a nearly new leading order mass matrix. However, by changing the TBM mass matrix with the bare minimum of self-sufficient characteristics, we can produce a vanishing reactor angle. The further implication of the current disturbance is that the atmospheric mixing angle $\boldsymbol{\theta_{23}}$ is likewise affected by similar parameters. In this way, we've demonstrated how the same parameter that creates a vanishing reactor angle $\boldsymbol{\theta_{13}}$ may also determine the atmospheric mixing angle's undetermined octant.

Conflict of Interest

Authors do not have any conflict of interest.

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