J. Integr. Sci. Technol. 2023, 11(2), 478 Article

Journal of Integrated
SCIENCE & TECHNOLOGY

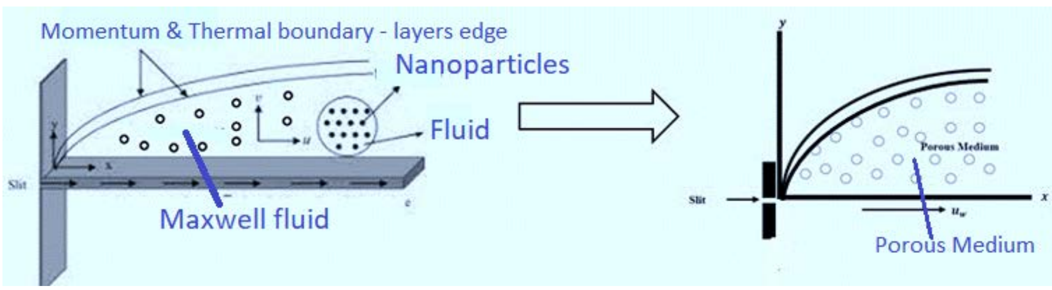
Numerical investigation of MHD nanofluid considering second-order velocity slip effect over a stretching sheet in porous media

Shefali Jauhri,[1*] Upendra Mishra,[2]

[1]Department of Mathematics, Jaipur National University Agra Bypass, Near New RTO Office, Jagatpura, 302017 Jaipur, India.
[2]Department of Mathematics, Amity University Rajasthan, NH-11 C, Kant Kalwar, Delhi-Jaipur Highway, Near Achrol Village, Jaipur, India

Received on: 24-Oct-2022, Accepted and Published on: 01-Dec-2022

ABSTRACT



This study investigates the flow, heat, and mass transport properties of Maxwell fluid over a stretched sheet of porous media. The partial differential conditions containing second-order slip boundary conditions can be transformed into nonlinear ordinary differential conditions using proper similitude adjustments. It is possible to acquire numerical solutions for a set of transformed equations that were created from a physical model using the Runge-Kutta method with MATLAB programming. The implications of several other non-dimensional properties are also being studied. In fluid flow, many applied slip models are defined by researchers, like slip model (first-order also known as Maxwell slip model, the 1.5-order slip model, the FK model, and the second-order slip model. The wall's slip velocity boundary condition is created by adding the bulk velocity expansion to the wall collision molecules' tangential momentum transfer rate, which is then matched to the area's wall shear stress.

Keywords: Maxwell fluid, Porous Stretching Sheet, 2nd Order Slip Boundary Conditions, Shooting Method

INTRODUCTION

Due to its industrial applications, fluid flow and heat transmission in porous media have garnered a lot of attention in recent years, like filtration, purification, blood flow models, bio-fluid, steel rolling, oil extraction, atomic power plants, the petroleum industry, and underground water resources, among others.[1] Also, in the petroleum and geothermal industries, the transport qualities of fluid-saturated porous materials are critical. Natural porous materials like soil and cracked rocks are settled with fluid, which migrates and transports the fluid through the material under the effect of local pressure gradients. Scientists and researchers are interested in porous media not only because of their structure but also because numerous methods are being used to improve their efficiency and rate of heat transfer.[2,3] One strategy for increasing the rate of heat transfer is to enhance fluid's conducting qualities. In fluid flow problems, there are two types of slip circumstances to consider: slip condition and no-slip condition. A no-slip boundary condition states, when no relative motion exists between the walls and the fluid. However, this condition does not apply to all surfaces.

Some surfaces, such as those affected by the flow of polymeric liquids, have liquids that slip against the walls. The slip condition, also known as the Navier condition, is extremely important in lubricating, medical-sciences, artificial heart valve polishing, and biological fluids. There was a lot of effort put forth by scholars on MHD fluid flow problems in slip situations. Ahmed et.al. explored the viscous flow problem of incompressible fluid on a stretched surface in a boundary layer.[4] He classified the variable thermal conductivity suction parameter's effect on the field of temperature into two components: the specified temperature of surface and the advised heat flux[5] of stretched surface. Aman et.al. investigated the influence of 2nd-order slip from MHD flow of a fractional Maxwell

Corresponding Author: Shefali Jauhri
Department of Mathematics, Jaipur National University Agra Bypass, Near New RTO Office, Jagatpura, 302017 Jaipur, India
Email: Shefali.jauhri3@gmail.com

Cite as: *J. Integr. Sci. Technol., 2023, 11(2), 478. URN:NBN:sciencein.jist.2023.v11.478*

ISSN: 2321-4635
http://pubs.thesciencein.org/jist

fluid on a moving plate, two numerical approaches are compared, and a semi-analytical solution using the Laplace transform is provided.[6] Bhargava et al. explained flow of micropolar fluid across a nonlinearly stretching-surface.[7] Using a stretching sheet, Choi was a pioneer in the use of nanoparticles in nanofluids, having done so in the 1980s.[8]

Cortell et.al. examined two cases of flow of viscous fluid across a non-linear stretching surface: the CST and the PST.[9] Under slip conditions, Fang et.al. investigated the flow of magnetohydrodynamic (MHD) on a linear stretched surface contained in the porous material.[10] Gangadhar et al. used computational approaches to investigate the effects of the influence of varied suction/injection and viscous dissipation on boundary layer flow. Gangadhar et al. used the spectrum relaxation method to analyse the characteristics of unsteady flow of nano-fluid boundary layer.[11,12]

An investigation into the flow of magnetic hydrodynamic, a Jeffrey liquid using radially nonlinear stretched surface have been done.[13] Newtonian and Joule heating are the two main types of heat transfer. An exponentially stretched sheet with slip conditions was studied by Hayat et al. in their study of unsteady MHD flow.[14] Jauhri and Mishra investigated the dual solution of boundary layer flow with mixed convection under second-order slip boundary conditions.[15] Nano fluid boundary layer flow across a nonlinear permeable stretching sheet in case of partial slip conditions predetermined ambient temperature was numerically explored.[16]

Kang et al. used to examine the enhancement of the thermal conductivity of nanofluid under a transient hot-wire, instead of the true volume the estimated volume of nanoparticles was employed.[17] Li Yi et al. studied the heat and mass transfer rates in MHD Williamson nanofluid flow across an exponentially porous surface by experimentation.[18] Two alternative heat transfer conditions have been analyzed: Prescribed exponential order heat flux (PEST) and prescribed exponential order surface temperature (PEST). Rafique et al. examined the effects of radiation and Soret on a tilted stretchy sheet, which is the focus of this research.[19] Brownian motion and thermophoretic effects are taken into account in Buongiorno's model.

Gangadhar et. al. investigated the study of, mass transfer, heat transfer qualities in boundary layer motion about a stretched surface in a porous material equipped with Al_2O_3 (water-based nanofluids) and TiO_2 (water), with variable suction or injection effects that were numerically examined.[20] Three types of flows in porous material can be classified using the Reynolds number. Inertial effects predominate in the transition between the Darcy and Forchheimer regimes, which occurs when the laminar and nonlinear regimes meet, creating a rough range described by Zeng.[21] The persistent MHD boundary layer zero motion point flow of an incompressible, dense, and electrically conducting liquid past an expanding/constricting sheet with the influence of an induced magnetic field is studied by Junoh.[22] Heat conductivity was covered by Kang et al in numerical and exploratory investigation of nanofluids.[17] Khanafer et al. analysed of the heat transfer behaviour of nanofluids inside a sealed space took high molecule scattering into account.[23] Numerous people believe that nanotechnology will be one of the key forces behind the next significant mechanical revolution of this century, after the work of these innovators. It refers to the crucial mechanical front line being investigation at the moment. It aims to regulate the issue's subatomic structure with the goal of development in essentially every industry and open undertaking, such as organic sciences, physical sciences, hardware cooling, transportation, the earth, and national security, among others. On a stretching sheet of nanofluid, Khan & Pop numerically examined the laminar fluid flow problem on a flat surface.[24] Over a stretching sheet, Kumaran et.al. studied on viscous incompressible flow.[25] He used a quadratic polynomial of the distance from the slit to represent the velocity in his paper, and the sheet is subject to a linear mass flux. Kuznetsov & Nield performed an analytical analysis of the flow of a nanofluid past a vertical plate in a normal convective boundary layer.[26] Moreover, describe the effects of thermophoresis and Brownian mobility. Malavandi et.al. studied various types of nanoparticles on a sheet that was contracting and expanding with a stagnation point flow.[27] According to Merkin an exothermic surface reaction may be affected by a stagnation-point stream on an extended or contracting surface.[28] The boundary parameter, which is an estimation, estimates the surface's velocity in relation to the external stream. Mishra et al[29] explored the "Axisymmetric" stream of a viscous incompressible liquid over a narrowing vertical surface while taking into account the boundary conditions of first-order heat slip and second-order momentum slip.[30] When there is a temperature difference between the sheet and the surrounding liquid, Mishra et al explanation of the boundary layer flow and heat transfer of an incompressible liquid along a vertical temperamental extending sheet in a tranquil liquid is introduced.[30] Myers et.al. has experimentally shown the impact of various heat & mass transport parameters on nanofluid.[31] Nadeem et al numerical study of the Maxwell fluid's heat transfer to a stretching sheet.[32] With the base surface of the sheet being warmed by convection from a hot liquid, Ramesh et.al. consistently generated a 2D boundary layer stream of a thick, dusty fluid across a stretching sheet.[33] The thermal effect of radiation and a magnetic field across an inclined vertical plate was studied by Reddy et al.[34] The equation of momentum for boundary layer flow across a stretching sheet was solved by Siddappa et.al. after researching the crane's flow problem in the visco-elastic fluid of Walter's liquid model.[35] A porous medium over an impenetrable stretched surface with resistive heating and heat production or absorption was explored by Subhas et.al. along with the features of visco-elastic fluid flow and heat transmission.[36] Wang looked into the crane's paper extension.[37] He researched the precise solution to Navier Stokes' condition while working on the three-dimensional fluid movement on a plane boundary stretching sheet.[37]

We want to concentrate on the impact of the previously unmentioned second-order velocity parameter (L_2), porosity parameter (k_p), Deborah number ($\dot{\beta}$),), N_t-thermophoresis parameter on natural convection heat transfer and linear flow over a stretched sheet.

Mathematical Formulation:

Consider incompressible, steady MHD flows in two dimensions with slip condition and a Maxwell fluid stretching surface across a permeable medium. The heat transfer process is analyzed by viscous dissipation. The surface along the x-axis and $y > 0$ is the confined region. The generated flow is linear, so the x-axis is subjected to two forces that are equal yet opposite one another.T_∞ and C_∞ are the ambient temperature and concentration.

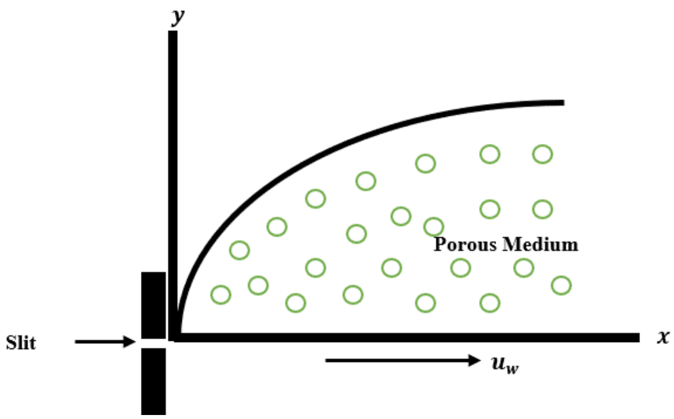


Figure 1. Diagrammatic depiction of fluid flow in porous sheet

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=u\frac{\partial u}{\partial x}+\nu_f\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{\rho k_0}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K_f}{(\rho c_p)_f}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Subject to the 'Boundary Conditions' are

$$\left.\begin{aligned} u &= c'ax+u_{(slip)} \\ T &= T_W+K_T\frac{\partial T}{\partial y}\end{aligned}\right\} as\ y=0 \tag{4}$$

$$\left.\begin{aligned} u &\to 0 \\ T &\to 0\end{aligned}\right\} as\ y\to\infty \tag{5}$$

Here, u (on x-direction) and v (on y-direction) are velocity components, 'α_m is the fluid's coefficient of thermal diffusion, 'υ' kinematic-viscosity, 'c'' is arbitrary constant, 'D_B' coefficient of Brownian diffusion and fluid stress parameter '$\tau'=\frac{(\rho c_p)_p}{(\rho c_p)_f}$', ratio between the adequate heat capacity of nanomaterial and fluid.

$u_{(slip)}$ is the slip velocity at the wall, which can be defined as,

$$u_{(slip)}=\frac{2}{3}\left(\frac{3-al^3}{a}-\frac{3}{2}\frac{(1-l^2)}{k_n}\right)\gamma\frac{\partial u}{\partial y}-\frac{1}{4}\left(l^4+\frac{2}{k_n^2}(1-l^2)\right)\gamma^2\frac{\partial^2 u}{\partial y^2}$$

$$u_{(slip)}=A_1\frac{\partial u}{\partial y}+A_2\frac{\partial^2 u}{\partial y^2}$$

Where, $l=min\left[\frac{1}{k_n},1\right]$, γ is molecular mean free path and 'a' is momentum coefficient with $0\le a\le 1$.Hence value of 'l' is $0\le l\le 1$. The second term in $u_{(slip)}$ condition is positive Since mean free path is always positive and $A_2<0$.

Transformed terms by using similarity variables with boundary conditions (1)-(4).

$$\eta=y\sqrt{\frac{a}{\nu_f}}$$

$$u=axf'(\eta) \text{ and } v=-\sqrt{a\nu_f}\,f(\eta) \tag{6}$$

The non-dimensional temperature and concentration term by using similarity variables are

$\theta(\eta)=\frac{T_f-T_\infty}{T_w-T_\infty}$,

$$T_f=T_\infty+A'x\,\theta(\eta)\text{and} \tag{7}$$

Using equation (6) and (7) in equations (2) and (3). Following differential equations obtained,

$$f'''+ff''+\mathcal{E}^2-f'^2+\dot{\beta}(2ff'f-ff^2)-k_pf'=0 \tag{8}$$

$$\frac{1}{\mathrm{Pr}}\left(1+\frac{4R_d}{3}\right)\theta''+f\theta'+\mathrm{Ec}(\mathrm{k_p}f'^2+\mathrm{f}''^2)=0 \tag{9}$$

Here

$P_R=\frac{\nu_f}{\alpha}$ is 'Prandtl-Number'

Transformed 'Boundary-Conditions' are

$$\left.\begin{aligned} f(0)&=S \\ f'(0)&=c+L_1f''(0)+L_2f'''(0) \\ \theta(0)&=1+\delta_1\theta'(0)\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} f'&\to 0 \\ \theta&\to 0\end{aligned}\right\} \text{as } \eta\to\infty \tag{11}$$

Here $L_1=\sqrt{\frac{a}{\nu_f}}$, $\delta_1=k_T\sqrt{\frac{a}{\nu_f}}$, $\delta_2=k_C\sqrt{\frac{a}{\nu_f}}$are nondimensional slip parameter and $S>0$is for suction and $S<0$ is for injection.

2.1 Nusselt Number: -Heat transfer across a fluid is characterised by the Nusselt number, which distinguishes between convectional and conductional modes of heat transfer. The Nusselt number is denoted by the following:

$$Nu=\frac{xq_w}{k(T_w-T_\infty)}$$

Here,

$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)$, stress along the plate's tangent.

$q_w=-k\left(\frac{\partial T}{\partial y}\right)$ at y=0.

k=heat conduction coefficient.

x=characteristic length.

With the help of Eq. (7) we get,$Re_x=\frac{u_wx}{\upsilon}$ (local Reynolds number). $\frac{Nu}{Re_x^{1/2}}=-\theta'(0)$ Reduced Nusselt number respectively.

Table 1: Comparison of the rate of heat transfer of wall $-\theta'(0)$ for Newtonian fluids with various Pr values when $\dot{\beta}=k_p=Ec$=0.

$\dot{\beta}$	k_p	$-f''(0)$	$-\theta'(0)$
0.1	0.1	1.1243	0.21373
0.2	0.1	1.1454	0.20654
0.3	0.1	1.1681	0.19286
0.4	0.1	1.1835	0.18998
0.5	0.1	1.2044	0.18876

Table 2: -Numerical Values of $-f''(0)$, $-\theta'(0)$ for different values of $\dot{\beta}$, k_p.

Pr	Khan & Pop (2010)	Malik et. al. (2017)	K. Gangadhar et.al (2019)	Present Result
0.7	0.4539	0.45392	0.45391616	0.45391
2	0.9113	0.91135	0.91135768	0.91136
7	1.8954	1.89543	1.89540326	1.89542
20	3.3539	3.35395	3.35390414	3.35394

Result and Discussion:

In this article, flow of Maxwell fluid across a stretching surface was examined in relation to the impacts of the 2nd-order slip parameter velocity. To solve the differential equations (8)-(9) with equations (10-11), we employed the Runge-Kutta fourth-order approach. Physical parameters (for example, the porous parameter, the 1st and 2nd order velocity slip parameters, the heat transfer slip parameter, and the Deborah number) have been extensively studied in order to fully comprehend their impact. A temperature and velocity profile graph and a local Nusselt number are used to depict the results of this study. To validate the method, the values of $-\theta'(0)$ obtained by the present system are compared to those published by Khan et al., Malik et al,[38] and K. Gangadhar et al. for various values of Pr when $\dot{\beta} = k_p = Ec$=0. Table 1 and Table 2 displays the comparison, which demonstrates an excellent match. Since the system has two solutions, they are referred to as the first solution and the second solution. It is clear from (11) and (12) that f "(0) and -θ'(0) measure the rate of heat transfer and skin-friction, respectively. In the boundary layer, f'(η) and $\theta(\eta)$ measure, respectively, the fluid velocity and temperature distribution Figures 2 and 3 show how the Deborah parameter '$\dot{\beta}$' influences dimension-less velocity and temperature-profile. Velocity increases, and the temperature profile increases at some point after that, then decreases. Figures 4(a) and 4(b) and 5(a) and 5(b) show that increasing the second-order slip parameter increases the slip velocity and temperature profile due to a reduction in drag force and frictional heat generation. Figures 6 and 7 illustrate the effect of the stretching and shrinking parameter 'S' on the non-dimensional temperature profile. It is noted that by increasing the value of 'S' while keeping the other variables constant, the temperature profiles decrease in magnitude. In figures 8 and 9, as the porosity parameter increased, the resistive force increased, generating friction heat; hence, the thermal profile increased while velocity decreased. Figure10 is illustrate Nusselt number effect on second order velocity slip parameter. Figures 11 and 12 illustrate the effect of the radiation parameter and Eckert number 'E_c' on the non-dimensional temperature profile. It is noted that by increasing the value of 'E_c' and R_d the temperature profile increases in magnitude.

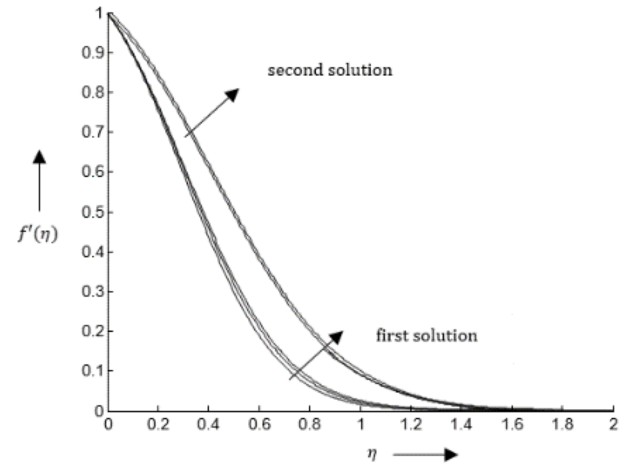


Figure 2 is velocity profile dual solution for different $\dot{\beta}$ =0.1,0.3,0.5 versus η with Pr=0.7,$k_p = .2, Ec$=0.1, $L_2 = 0.1$

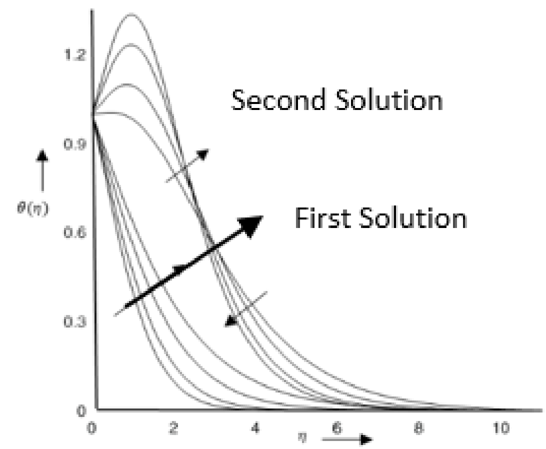


Figure 3 dual solution of temperature profile for different $\dot{\beta}$ =0.2,0.4,0.6,0,8versus η with Pr=0.7,$k_p = .2, Ec$=0.1,$L_2 = .1$

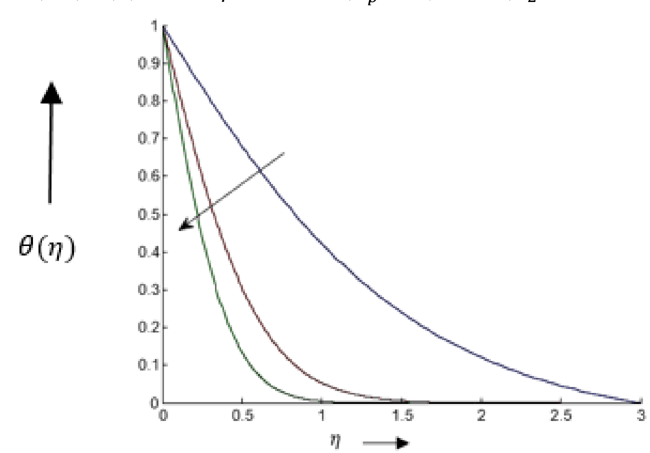


Figure 4(a) First Solution

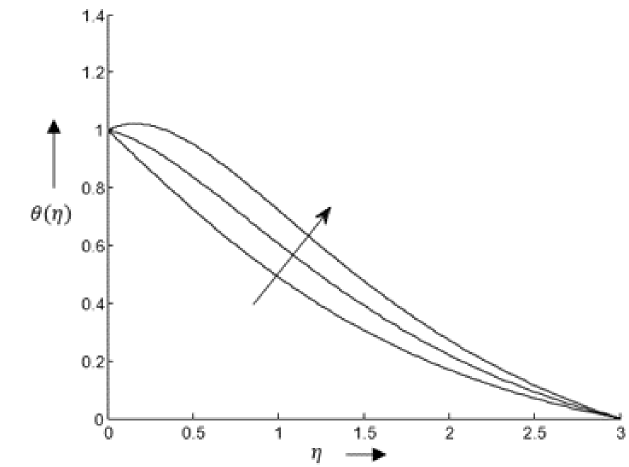


Figure 4(b) Second Solution

Figure 4(a) and Figure4(b) represent dual solution of velocity profile for different values of L_2 =0.1,0.3,0.5 versus η with Pr=0.7,$k_p = .2, Ec$=0.1, $\dot{\beta}$=0.1.

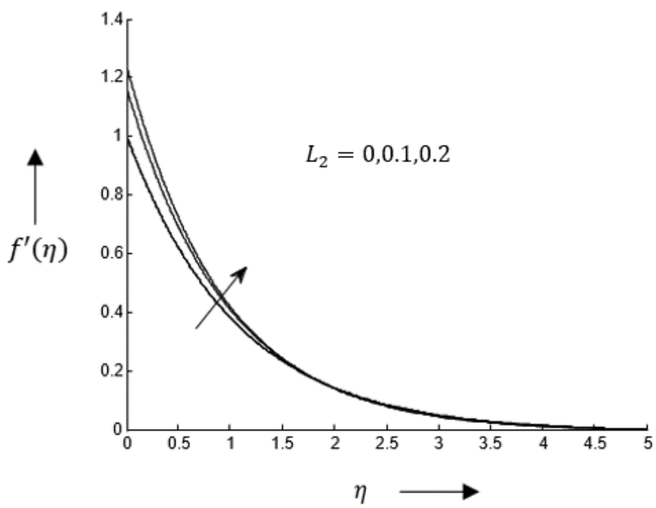


Figure 5(a) First Solution

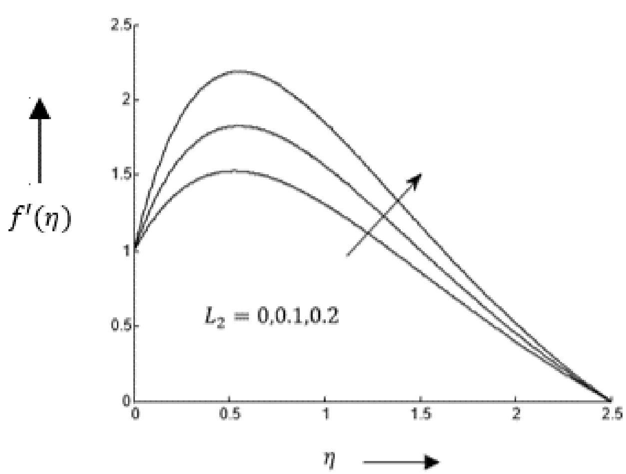


Figure 5(b) Second Solution

Figure 5(a). and 5(b). represent velocity profile dual solution for various L_2versus η with Pr=0.7,$k_p = .2, Ec$=0.1, $\dot{\beta}$=0.1

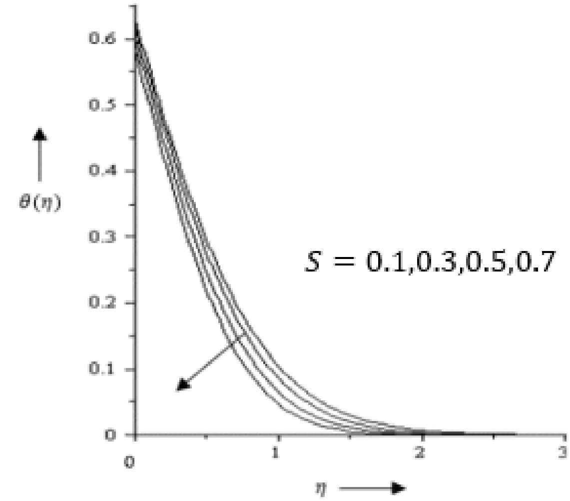


Figure 6. temperature profile for various values of $S(> 0\ )$versus η with Pr=0.7,$L_2 = .1, Ec$=0.1, $\dot{\beta}$=0.1.

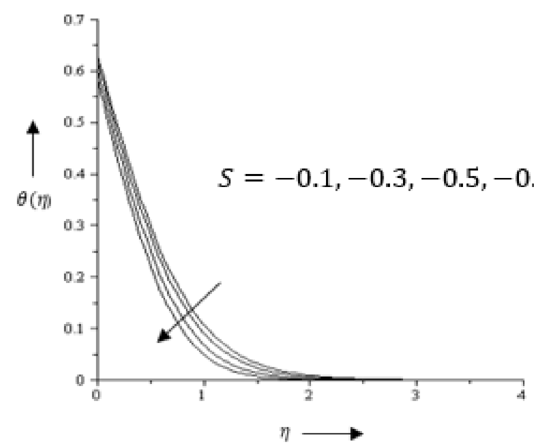


Figure 7. Temperature profile for various values of $S(< 0)$versus η with Pr=0.7,$L_2 = .1, Ec$=0.1, $\dot{\beta}$=0.1.

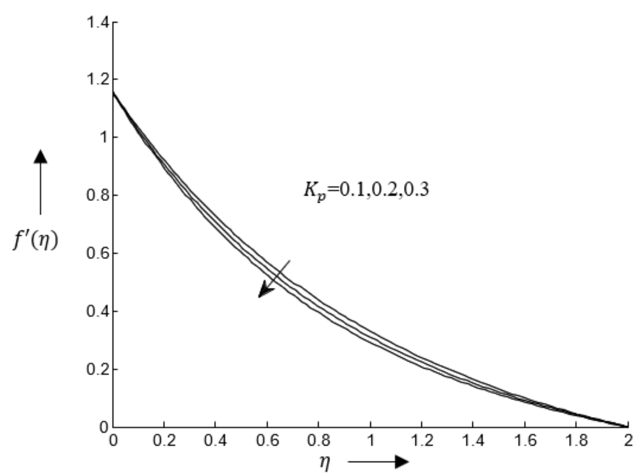


Figure 8. velocity profile for various values of $k_p$versus $\eta$with Pr=0.7, $L_2 = .1, Ec$=0.1, $\dot{\beta}$=0.1.

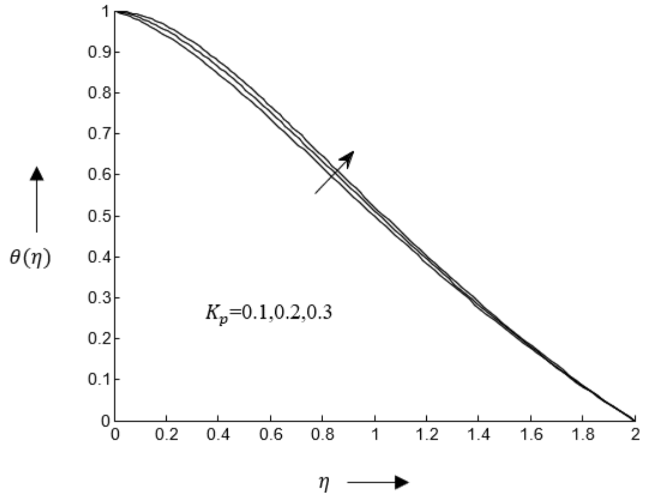


Figure 9. temperature profile for various values of $k_p$versus η with Pr=0.7,$L_2 = .1, Ec$=0.1, $\dot{\beta}$=0.1.

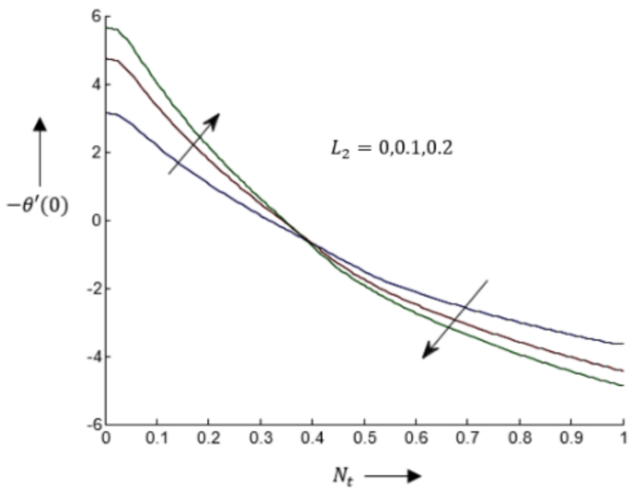


Figure 10. the variations in Nusselt numberL_2 versus N_t η with Pr=0.7,$L_2 = .1, Ec$=0.1, $\dot{\beta}$=0.1

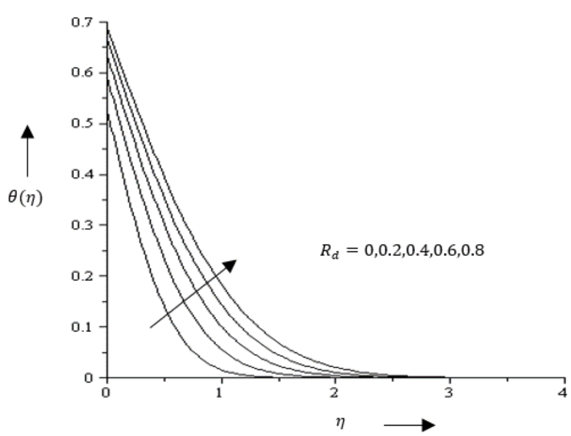


Figure 11. temperature profile for various values of $R_d$versus η with Pr=0.7,$L_2 = .1, Ec$=0.1, $\dot{\beta}$=0.1.

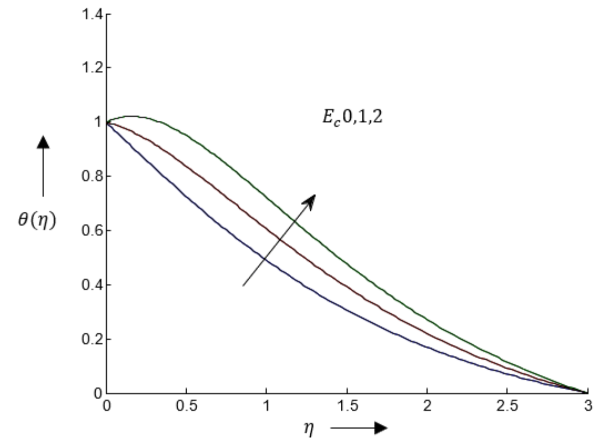


Figure 12. Temperature profile for various values of $E_C$versus η with Pr=0.7,$L_2 = .1, k_p$=0.1, $\dot{\beta}$=0.1.

Conclusion

The Runge-Kutta method is used to examine the boundary layer flow of Maxwell fluid over a stretching surface in the presence of a porous media. The data is presented in graphs and tables, and the findings are consistent with previous research. Deborah number $\dot{\beta}$, porosity parameter k_p, Eckert number Ec, Prandtl number Pr are all investigated in detail. Some of the most important findings from the aforementioned analysis are listed below.

The following are some of the analysis's significant findings:

I. As Deborah's number and the second order slip parameter increase, velocity profiles increases, whereas velocity decreases as k_p increase.
II. The temperature profile rises in the first solution and decreases in the second solution as the L_2, $S > 0, S < 0$ and increases as $k_p$increases.
III. The temperature profile rises as R_d and E_c increases.
IV. N_t rises in first-half and decreases in second half as L_2, increases.

Conflict of Interest

Author declared no conflict of interest of any kind for publication of the article.

References and Notes

1. P. Surawattanawan, J. Chutikusol. Mathematical modeling and optimum design for capillary tubes in R-410A Air Conditioner. *J. Integr. Sci. Technol.* **2023**, 11 (1), 403.
2. R. Kumar, M.P. Chaudhary, M.A. Shah, K. Mahajan. A mathematical elucidation of separation membrane operations and technology of chemical and physical processes: An advanced review. *J. Integr. Sci. Technol.* **2020**, 8 (2), 57–69.
3. R. Kumar, K. Mahajan, C.A. Igwegbe, et al. Chemical engineering of separation membrane, interfacial strategies, and mathematical modeling: a thorough analysis. *J. Integr. Sci. Technol.* **2021**, 9 (2), 75–84.
4. N. Ahmad, N. Khan. Boundary layer flow past a stretching plate with suction and heat transfer with variable conductivity. *Indian J. Eng. Mater. Sci.* **2000**, 7 (1), 51–53.
5. K. Govardhan, S. Muthuraja, A.N. Grace. Multiphysics modeling and optimisation of gas flow characteristics in a novel flow metric based gas sensing chamber with integrated heater. *J. Mater. Nanosci.* **2022**, 9 (2), 138–146.

6. S. Aman, Q. Al-Mdallal, I. Khan. Heat transfer and second order slip effect on MHD flow of fractional Maxwell fluid in a porous medium. *J. King Saud Univ. - Sci.* **2020**, 32 (1), 450–458.
7. R. Bhargava, S. Sharma, H.S. Takhar, O.A. Bég, P. Bhargava. Numerical Solutions for Micropolar Transport Phenomena over a Nonlinear Stretching Sheet. *Nonlinear Anal. Model. Control* **2007**, 12 (1), 45–63.
8. S.U.S. Choi, Z.G. Zhang, W. Yu, F.E. Lockwood, E.A. Grulke. Anomalous thermal conductivity enhancement in nanotube suspensions. *Appl. Phys. Lett.* **2001**, 79 (14), 2252–2254.
9. R. Cortell. Viscous flow and heat transfer over a nonlinearly stretching sheet. *Appl. Math. Comput.* **2007**, 184 (2), 864–873.
10. T. Fang, J. Zhang, S. Yao. Slip MHD viscous flow over a stretching sheet - An exact solution. *Commun. Nonlinear Sci. Numer. Simul.* **2009**, 14 (11), 3731–3737.
11. K. Gangadhar. Radiation and viscous dissipation effects on laminar boundary layer flow nanofluid over a vertical plate with a convective surface boundary condition with suction. *J. Appl. Fluid Mech.* **2016**, 9 (4), 2097–2103.
12. K. Gangadhar, T. Kannan, G. Sakthivel, K. DasaradhaRamaiah. Unsteady free convective boundary layer flow of a nanofluid past a stretching surface using a spectral relaxation method. *Int. J. Ambient Energy* **2020**, 41 (6), 609–616.
13. T. Hayat, W.A. Khan, S.Z. Abbas, S. Nadeem, S. Ahmad. Impact of induced magnetic field on second-grade nanofluid flow past a convectively heated stretching sheet. *Appl. Nanosci.* **2020**, 10 (8), 3001–3009.
14. T. Hayat, A. Shafiq, A. Alsaedi, S.A. Shahzad. Unsteady MHD flow over exponentially stretching sheet with slip conditions. *Appl. Math. Mech. (English Ed.* **2016**, 37 (2), 193–208.
15. S. Jauhri, U. Mishra. Dual Solutions of EMHD Nanofluid at Stretching Sheet with Mixed Convection Slip Boundary Condition. *Int. J. Heat Technol.* **2021**, 39 (6), 1887–1896.
16. K. Das. Nanofluid flow over a non-linear permeable stretching sheet with partial slip. *J. Egypt. Math. Soc.* **2015**, 23 (2), 451–456.
17. H.U. Kang, S.H. Kim, J.M. Oh. Estimation of thermal conductivity of nanofluid using experimental effective particle volume. *Exp. Heat Transf.* **2006**, 19 (3), 181–191.
18. Y.-X. Li, M.H. Alshbool, Y.-P. Lv, et al. Heat and mass transfer in MHD Williamson nanofluid flow over an exponentially porous stretching surface. *Case Stud. Therm. Eng.* **2021**, 26, 100975.
19. K. Rafique, H. Alotaibi, T.A. Nofal, et al. Numerical Solutions of Micropolar Nanofluid over an Inclined Surface Using Keller Box Analysis. *J. Math.* **2020**, 2020, 1–13.
20. K. Gangadhar, T. Kannan, K. DasaradhaRamaiah, G. Sakthivel. Boundary layer flow of nanofluids to analyse the heat absorption/generation over a stretching sheet with variable suction/injection in the presence of viscous dissipation. *Int. J. Ambient Energy* **2020**, 41 (9), 969–980.
21. Z. Zeng, R. Grigg. A Criterion for Non-Darcy Flow in Porous Media. *Transp. Porous Media* **2006**, 63 (1), 57–69.
22. M.M. Junoh, F.M. Ali, N.M. Arifin, N. Bachok, I. Pop. MHD stagnation-point flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid with induced magnetic field. *Int. J. Numer. Methods Heat Fluid Flow* **2020**, 30 (3), 1345–1364.
23. K. Khanafer, K. Vafai, M. Lightstone. Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *Int. J. Heat Mass Transf.* **2003**, 46 (19), 3639–3653.
24. W.A. Khan, I. Pop. Boundary-layer flow of a nanofluid past a stretching sheet. *Int. J. Heat Mass Transf.* **2010**, 53 (11–12), 2477–2483.
25. V. Kumaran, G. Ramanaiah. A note on the flow over a stretching sheet. *Acta Mech.* **1996**, 116, 229–233.
26. A. V. Kuznetsov, D.A. Nield. Natural convective boundary-layer flow of a nanofluid past a vertical plate. *Int. J. Therm. Sci.* **2010**, 49 (2), 243–247.
27. A. Malvandi, F. Hedayati, D.D. Ganji. Nanofluid flow on the stagnation point of a permeable non-linearly stretching/shrinking sheet. *Alexandria Eng. J.* **2018**, 57 (4), 2199–2208.
28. J.H. Merkin, I. Pop. Stagnation point flow past a stretching/shrinking sheet driven by Arrhenius kinetics. *Appl. Math. Comput.* **2018**, 337, 583–590.
29. P. Kaushik, U. Mishra. Numerical study due to mixed convection nanofluid flow with the effect of velocity slip and thermal conductivity across curved stretching surface. *J. Integr. Sci. Technol.* **2022**, 10 (2), 73–78.
30. U. Mishra, G. Singh. Dual solutions of mixed convection flow with momentum and thermal slip flow over a permeable shrinking cylinder. *Comput. Fluids* **2014**, 93, 107–115.
31. T.G. Myers, H. Ribera, V. Cregan. Does mathematics contribute to the nanofluid debate? *Int. J. Heat Mass Transf.* **2017**, 111, 279–288.
32. S. Nadeem, R.U. Haq, Z.H. Khan. Numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles. *J. Taiwan Inst. Chem. Eng.* **2014**, 45 (1), 121–126.
33. G.K. Ramesh, B.J. Gireesha, R.S.R. Gorla. Boundary layer flow past a stretching sheet with fluid-particle suspension and convective boundary condition. *Heat Mass Transf. und Stoffuebertragung* **2015**, 51 (8), 1061–1066.
34. P. Sudarsana Reddy, A.J. Chamkha, A. Al-Mudhaf. MHD heat and mass transfer flow of a nanofluid over an inclined vertical porous plate with radiation and heat generation/absorption. *Adv. Powder Technol.* **2017**, 28 (3), 1008–1017.
35. B. Siddappa, S. Abel. Non-Newtonian flow past a stretching plate. *ZAMP Zeitschrift für Angew. Math. und Phys.* **1985**, 36 (6), 890–892.
36. A. Subhas, P. Veena. Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet. *Int. J. Non. Linear. Mech.* **1998**, 33 (3), 531–540.
37. C.Y. Wang. The three-dimensional flow due to a stretching flat surface. *Phys. Fluids* **1984**, 27 (8), 1915–1917.
38. A. Hussain, M.Y. Malik, M. Awais, T. Salahuddin, S. Bilal. Computational and physical aspects of MHD Prandtl-Eyring fluid flow analysis over a stretching sheet. *Neural Comput. Appl.* **2019**, 31, 425–433.